5) |

unit - I : Basic Probability

short questions

1) Twelve Balls are distributed at random among three boxes what is the Probability that first Box will contain 3 Balls?

2) Three students A, B, C are in running race. A and B have the same Probability of winning and each is twice as likely to win as C. Find Probability that B or C wins.

(JNTU April 2006, (A) NOV. 2010)

3) Calculate expectation and variance of x, if Probability distribution of Random variable x is given by.

x	-1	0	1	2	3
f	0.3	0.1	0.1	0.3	0.2

(JNTU 2006 (set NO: 2)

4) Find Mean and variance of uniform Probability distribution given by $f(x) = \frac{1}{n}$ for $x = 1, 2, 3, \dots h$

(JNTU 2001, NOV 2009, NOV. 2010, DEC 2011)

5) A sample of 4 Items is selected at Random from Box containing 12 items of which 5 are defective. Find expected number E of defective Items.

(JNTU (H) NOV. 2009 (set NO: 4), (A) NOV. 2010)

6) If Random variable has Probability density $f(x)$ as

$f(x) = \begin{cases} 2e^{-2x}, & \text{for } x > 0 \\ 0, & \text{for } x \leq 0 \end{cases}$ find Probabilities that it will Take on

value (i) between 1 and 3 (ii) greater than 0.5.

(JNTU) 2001, 2006S (Set NO:4), H III yr Nov. 2015).

7) Find value of k and distribution function $f(x)$ given the Probability density function of random variable x as:

$$f(x) = \frac{k}{x^2+1}, \quad -\infty < x < \infty$$

(JNTU(K) March 2014)(Set NO:3))

8) The first three Moments of distribution about value 2 of variable are 1, 16 and -40. Show that Mean = 3, variance = 15, and $\mu_3 = -86$.

9) If Probability that communication system will have High fidelity is 0.81 and Probability that it will have High fidelity and selectivity is 0.18. what is Probability that a system with High fidelity will also have High selectivity?

10) A fair die is Tossed twice. Find Probability of getting a 4, 5 or 6 on first Toss and 1, 2, 3 or 4 on second Toss.

(JNTU (K) 2009 (Set NO:)

Long questions

1) Two Marbles are drawn in succession from Box containing 10 red, 30 white, 20 Blue and 15 orange Marbles with replacement being made after each draw. Find Probability that
   i) Both are white
   ii) First is red and second is white.

2) Three Machines I, II, III Produce 40%, 30%, 30% of Total number of Items of factory. The Percentages of defective Items of those Machines are 4%, 2%, 3%. If an Item is selected at Random, find Probability that an item is defective.

3) A Businessman goes to Hotels x, y, z, 20%, 50%, 30% of the Time respectively. It is known that 5%, 4%, 8% of rooms in x, y, z have faculty Plumbings. what is Probability that Business man's room Having faculty Plumbing is assigned to Hotel z?

4) Two dice are thrown. let x assign to each Point (a,b) in s the maximum of its numbers ie $x(a,b) = Max(a,b)$ Find Probability distribution. x is random variable with $x(s) = \{1,2,3,4,5,6\}$. also find Mean and variance of distribution.

x	1	2	3	4	5	6
$P(x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

<div align="right">(JNTU (H) apr. 2012 (Set No:1)).</div>

5) A random variable x has following Probability function.

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(i) determine k (ii) evaluate $P(x<6)$, $P(x\geq6)$, $P(0<x<5)$ and $P(0\leq x\leq4)$ (iii) if $P(x\leq k) > (1/2)$ find minimum value of k and (iv) determine distribution function of x (v) Mean (vi) variance

<div align="center">(JNTU O4S, O8S, (A) NOV·11 (H) DEC·11 (K) May 10S, NOV· 2012 May 2014.</div>

6) A fair die is tossed. let Random variable x denote twice number appearing on die.

     (i) write Probability distribution of x

     (ii) The Mean   (iii) The variance

<div align="right">(JNTU (H) apr 2012)</div>

7) A continuous random variable has Probability density function

$$f(x) = \begin{cases} kx\,e^{-\lambda x}, & \text{for } x\geq0, \lambda>0 \\ 0, & \text{otherwise} \end{cases}$$

determine (i) k, (ii) Mean (iii) variance

<div align="right">(JNTU (A) DEC 2009, NOV·2010, DEC 2011, (H) May 2011, NOV 2012, May2013).</div>

8) A Continous random variable x has distribution function

$$f(x) = \begin{cases} 0, & \text{if } x \leq 1 \\ k(x-1)^4, & \text{if } 1 < x \leq 3 \\ 1, & \text{if } x > 3 \end{cases}$$

Determine (i) $f(x)$ (ii) k (iii) Mean   (JNTU 2004S, 2007S +Px.2012)

9) If x is continous random variable and k is constant then
Prove that (i) var $(x+k)$ = var (x) (ii) var (kx) = k^2 var (x)

(JNTU 2006, 2007) (Set NO: 1)

10) Find constant k such that

$$f(x) = \begin{cases} kx^2 & \text{if } 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

is Probability function
i) Find distribution function $f(x)$
ii) $P(1 < x \leq 2)$.

(JNTU NOV. 2008 (Set NO. 2)).

Objective Type Questions

1) A couple has 2 children. Probability Both children are Boys if older one is Boy is

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 1.

2) A card is drawn from ordinary Pack. The Probability that it is king given that it is face card is

(A) $\frac{1}{3}$ (B) $\frac{1}{13}$ (C) $\frac{4}{13}$ (D) $\frac{1}{2}$

3) Two dice are thrown simultaneously. The Probability that Product of numbers appearing on Top faces of dice is Perfect square is

(A) $\frac{1}{9}$ (B) $\frac{2}{9}$ (C) $\frac{1}{3}$ (D) $\frac{4}{9}$

4) The Mean square of zero mean random Process is $\frac{kT}{c}$ where k is Boltzmann's Temperature and c is Capatiance. standard deviation of Random Process is

(A) $\frac{kT}{c}$ (B) $\frac{\sqrt{kT}}{c}$ (C) $\sqrt{\frac{kT}{c}}$ (D) $\frac{c}{kT}$

5) The function $P(x)$ is given by $P(x) = \frac{A}{x^{\mu}}$ where A and μ are constants with $\mu > 1$ and $1 \le x < \infty$ and $P(x) = 0$ for $-\infty < x < 1$. For $P(x)$ to be Probability density function, value of A should be equal to,

(A) $\mu - 1$ (B) $\mu + 1$ (C) $\frac{1}{\mu - 1}$ (D) $\frac{1}{\mu + 1}$

Fill in the Blanks

1) Maximum value of Probability is ____.

2) The Mean of Probability distribution of number of Heads obtained in two flips of Balanced coin is ____.

3) The Mean of Probability distribution of numbers on face of die in throwing a die is ____.

4) If x is uniformly distributed in (a,b) then $E(x) =$ ____

5) The relation between Probability density function and cummulative density function of random variable is ____.

6) A Probability density function on interval $(a, 1)$ given by $\frac{1}{x^2}$ and outside this interval value of function is zero. The value of a is ____.

7) If x and y are random variables such that $E(2x+y) = 0$ and $E(x+2y) = 33$, then $E(x) + E(y) =$ ____.

Unit-II (Chapter-3)
Probability Distributions

Short Answers :-

1. A fair coin is tossed six times. Find the Probability of getting foor heads

2. If 3 of 20 tyres are defective and 4 of them are randomly chosen from inspection, what is the Probability that only one of the defective tyre will be included ?

3. Determine the binomial distribution for which the mean is 4 and variance 3

4. Six dice are thrown 729 times. How many times do you except at least three dice to show a 5 or 6?

5. A die is tossed thrice. A success is getting 1 (or) 6 on a toss. Find the mean and variance of the number of successes.

6. The Probability of a man hitting a target is $1/3$

   i. If he fires 5 times, what is the Probability of his hitting the target at least twice ?

   ii. How many times must he fires so that the probability of his hitting the target at least once is more than 90% ?

7. The Probability that a man hitting a target is $1/3$. If he fires 6 times, find the probability that he fires

   i. At the most 5 times ii. Exactly one iii. At least two times

8. If a bank received on the average 6 bad cheques per day, find the probability that it will receive 4 bad cheques on any given day.

9. Average number of accidents on any day on a national highway is 1.6
   i. at least one   ii. at most one

10. Prove that the three successive values of a Poisson variate cannot have equal probability of success.

Long - Answers :-

1. Mean of the binomial distribution.

2. Variance of the binomial distribution.

3. Ten coins are thrown simultaneously. Find the probability of getting at least
   i. Seven heads   ii. Six heads   iii. One head

4. Fit a binomial distribution to the following frequency distribution

x	0	1	2	3	4	5	6
f	13	25	52	58	32	16	4

5. Four coins are tossed 160 times. The number of times 'x' heads occurs is given below.

x	0	1	2	3	4
No. of times	8	34	69	43	6

Fit a binomial distribution to this data on the hypothesis that coins are unbiassed.

Fill in the blanks :-

1.) The Probability of getting 2 heads in toring 5 coins is ___

Ans:- 5/256

2.) The Probability of having at least one tail in four throws with a coin is ___

Ans:- 15/16

3.) If mean of the binomial distribution is 4 and Variance is 2 then P= ___

Ans:- ½

4.) A coin is tossed 3 times. The Probability of obtaining two heads will be ___

Ans:- 3/8

5.) The Probability of getting four heads in six tosses of a fair coin is ___

Ans:- 15/64

6.) The mean, median and mode of a normal distribution are ___

Ans:- Zero

7.) If $\mu = 5$ and $\sigma = 2$, the equation of the normal distribution is ___

Ans:- $f(x) = \dfrac{1}{2\sqrt{2\pi}} \cdot e^{\frac{(x-5)^2}{8}}$

6. If a random Variable has a Poisson distribution such that $P(1) = P(2)$, find i. Mean of the distribution
   ii. $P(4)$ iii. $P(a \geq 1)$ iv. $P(1 < a < 4)$.

7. Suppose 2% of the people on the average are left handed. Find i). the Probability of finding 3 or more lefthanded
   ii). the Probability of finding none or one left handed.

8. Given that $P(a=2) = 9P(a=4) + 90P(a=6)$ for a Poisson Variate X. find:
   i. $P(a<2)$ ii. $P(a>4)$ iii. $P(a \geq 1)$

9. Fit a Poisson distribution for the following data and Calculate the expected frequencies.

x	0	1	2	3	4
$f(x)$	109	65	22	3	1

10. In a normal distribution, 7% of the items are under 35 and 89% are under 63. Determine the mean and Variance of the distribution.

8). A fair coin is tossed n times. The probability that the difference between the number of heads and tails is $(n-3)$ is ( )

A. 2^{-n}

B. 0

C. $^nC_{n-3}2^{-n}$

D. 2^{-n+3}

Ans:- B

9). The Probability density function of a continuous random variable distributed uniformly between x and y (for $y > x$) is ( )

A. $\dfrac{1}{y-x}$

B. $\dfrac{1}{x-y}$

C. $x-y$

D. $y-x$

Ans:- A

10) If a random variable x has a poisson distribution with mean 5, then the expectation $E[(x+2)^2]$ equals : _____

Ans:- 54